Competitive equilibrium and the double auction

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Introduction

If you want to make a first-class economist, catch a parrot and teach him to say 'supply and demand' in response to every question you ask him. What determines wages? Supply and demand. What determines interest? Supply and demand. What determines the distribution of wealth? Supply and demand.

A joke from 'a long time ago' quoted in Irving Fisher's 'Introduction to Economic Science' (1910).

Introduction

More than 100 years later, the idea that supply equals demand ('competitive equilibrium') remains pervasive throughout economic theory:

- General equilibrium theory (Arrow and Hahn, 1971) with applications to macroeconomics (Lucas, 1977; Kydland and Prescott, 1982; …)
- Partial equilibrium models of discrimination (Becker, 1957), marriage markets (Becker, 1973), location choice (Glaeser, 2007), etc.
- Etc. etc.

Introduction

In part, the pervasiveness of competitive equilibrium (CE) may be due to the perception that its predictions have been experimentally vindicated by a series of double auction experiments starting with Smith (1962):

- Plott (1982): “The overwhelming result [from oral DA experiments] is that these markets converge to the competitive equilibrium even with very few traders”.
- Lin et al. (2020): “The results from simple competitive buyer–seller trading appear to be as close to a culturally universal, highly reproducible outcome as one is likely to get in social science, [at least] for young adults in ‘WEIRD’ societies. CE convergence in small markets should be considered as reproducible as the kinds of experiments that are done in a college chemistry laboratory to demonstrate universal chemistry principles . . . ”

Introduction

Existing experimental results appear to be surprisingly robust to

- Changing the number of bidders (Smith, 1965)
- Changing the shape of supply/demand curves (Smith and Williams, 1983)
- Changing the cultural context (Kachelmeier and Shehata, 1992)

Thus, while it may be possible to at least slow convergence to CE through large changes to the underlying environment — e.g. by allowing for resale as in Dickhaut, Lin , Porter, and Smith (2012) — the existing literature suggests fast convergence to CE in DA environments similar to those first considered by Smith (1965).

Introduction

In this paper, we revisit this conclusion

- Starting point: the CE model can make highly counterintuitive (but previously unnoticed) predictions
- To illustrate, suppose that there are 99 buyers, each with unit demand and with valuations £1.01, £2.01, £3.01, . . ., up to £99.01. Suppose that there are 99 potential sellers, each possessing one unit, and with valuations £0.99, £1.99, £2.99, . . ., up to £98.99. Then the (essentially unique) CE price is…

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- We will ask if counterintuitive predictions like these are borne out by the data

Introduction

Outline

- Section 2 formalises the idea of ‘CE preserving shifts’
- Section 3 shows experimentally that such shifts do change the price, at least in experiments with stationary value distributions
- Section 4 examines which models can explain this effect
- Section 5 studies whether the counterexample “disappears” if traders are allowed to drop out of the market as trade progresses

CE preserving shifts

- For simplicity, consider a unit mass of buyers indexed by $i \in [0,1]$
- Each buyer has a valuation $v_i \in [0,\bar{v_b}]$ where $\bar{v_b} > 0$ is the maximum buyer valuation. They choose to buy one unit of the good if and only if $v_i \geq p$
- The distribution of buyer valuations is described by the cumulative distribution function $F: \mathbb{R} \to [0,1]$. We assume that (1) F has full support on the interval $[0,\bar{v_b}]$ (2) F is continuous
- Let $d(p)$ denote market demand at price p. Then

  $$d(p) = \int_0^1 1(v_i \geq p)di = P(v_i \geq p) = 1 - F(p)$$

  where $1(v_i \geq p)$ is an indicator function

CE preserving shifts

Since $d(p) = 1 - F(p)$, observe that

- $d(0) = 1 - F(0) = 1$
- $d(\bar{v_b}) = 1 - F(\bar{v_b}) = 0$
- d is continuous (since F is continuous)
- d is strictly decreasing over the interval $[0, \bar{v_b}]$ (since F has full support on this interval, so is strictly increasing over the interval)

CE preserving shifts

- Symmetrically, there is a unit mass of sellers, indexed by $i \in [0,1]$
- Each seller has a valuation $v_i \in [0,\bar{v}_s]$ where $\bar{v}_s > 0$ is the maximum seller valuation. They choose to sell one unit of the good if and only if $v_i \leq p$
- The distribution of seller valuations is described by the cumulative distribution function $G\colon \mathbb{R} \to [0,1]$. We assume that (i) G has full support on the interval $[0,\bar{v}_s]$ (ii) G is continuous
- Let $s(p)$ denote market supply at price p. Then
$$s(p) = \int_0^1 1(v_i \leq p)di = P(v_i \leq p) = G(p)$$

where $1(v_i \leq p)$ is again an indicator function

CE preserving shifts

Since $s(p) = G(p)$, observe that

- $s(0) = G(0) = 0$
- $s(\bar{v}_s) = G(\bar{v}_s) = 1$
- s is continuous (since G is continuous)
- s is strictly increasing over the interval $[0, \bar{v}_s]$ (since G has full support on this interval)

CE preserving shifts

A *competitive equilibrium price* is a price $p^* \in \mathbb{R}^+$ such that $d(p^*) = s(p^*)$.

Observation: If valuations are distributed according to F and G, there is exactly one competitive equilibrium price.

CE preserving shifts

We now define the central concept of the section.

Definition 1. A *competitive equilibrium preserving demand contraction* is a transformation $T_b \colon [0, \bar{v}_b] \to \mathbb{R}^+$ such that

- $T(V_b) \leq V_b$ for all $V_b \leq p^* - \epsilon^-$
- $T(V_b) = V_b$ for all $V_b \in (p^* - \epsilon^-, p^* + \epsilon^+)$
- $T(V_b) \in [p^* + \epsilon^+, V_b]$ for all $V_b \geq p^* + \epsilon^+$

for some (small) $\epsilon^+, \epsilon^- > 0$.

CE preserving shifts

CE preserving shifts

Definition 2. A *competitive equilibrium preserving supply expansion* is a transformation $T_s \colon [0, \bar{v}_s] \to \mathbb{R}^+$ such that

- $T(V_s) \leq V_s$ for all $V_s \leq p^* - \epsilon^-$
- $T(V_s) = V_s$ for all $V_s \in (p^* - \epsilon^-, p^* + \epsilon^+)$
- $T(V_s) \in [p^* + \epsilon^+, V_s]$ for all $V_s \geq p^* + \epsilon^+$

for some (small) $\epsilon^+, \epsilon^- > 0$.

CE preserving shifts

CE preserving shifts

Proposition. Let p^* denote the CE price when buyer and seller valuations are distributed according to V_b and V_s respectively. Then if T_b (respectively, T_s) is a competitive equilibrium preserving demand contraction (supply expansion), p^* remains the unique competitive equilibrium price when buyer and seller valuations are distributed according to $V_b' = T_b(V_b)$ and $V_s' = T_s(V_s)$.

Experimental design

Basic idea: check if CE preserving shifts do indeed alter observed prices in double auctions

As usual, trading is quite unstructured:

- At any time, buyers can make bids or accept offers
- At any time, sellers can make asks or accept bids

Experimental design

In contrast to standard DA experiments, I use a 'queue' to keep supply/demand curves stationary: every time a trader drops out, a new trader with the exact same valuation enters the market (similar in spirit to Brewer, Huang, Nelson and Plott 2002). This has two advantages:

1. Probably more realistic than the classic set up: in real markets, trade does not stop after a couple of periods.

2. More importantly, it ensures that the CE remains fixed over time, allowing us to rigorously study whether the experimental market converges to the CE price. (In contrast, standard experiments allow the CE to fluctuate over time if trade does not exactly follow the "Marshallian path").

Experimental design

There are two treatments:

Symmetric Treatment: buyer valuations are 12, 32, 52, 72, 92; seller valuations are 8, 28, 48, 68, 88. [CE prices range from 48 to 52.]

Low Values Treatment: buyer valuations are 0, 0, 52, 52, 52; seller valuations are 0, 0, 48, 52, 52. [CE prices still range from 48 to 52.]

To control for subject fixed effects, I conduct both auctions within each session.

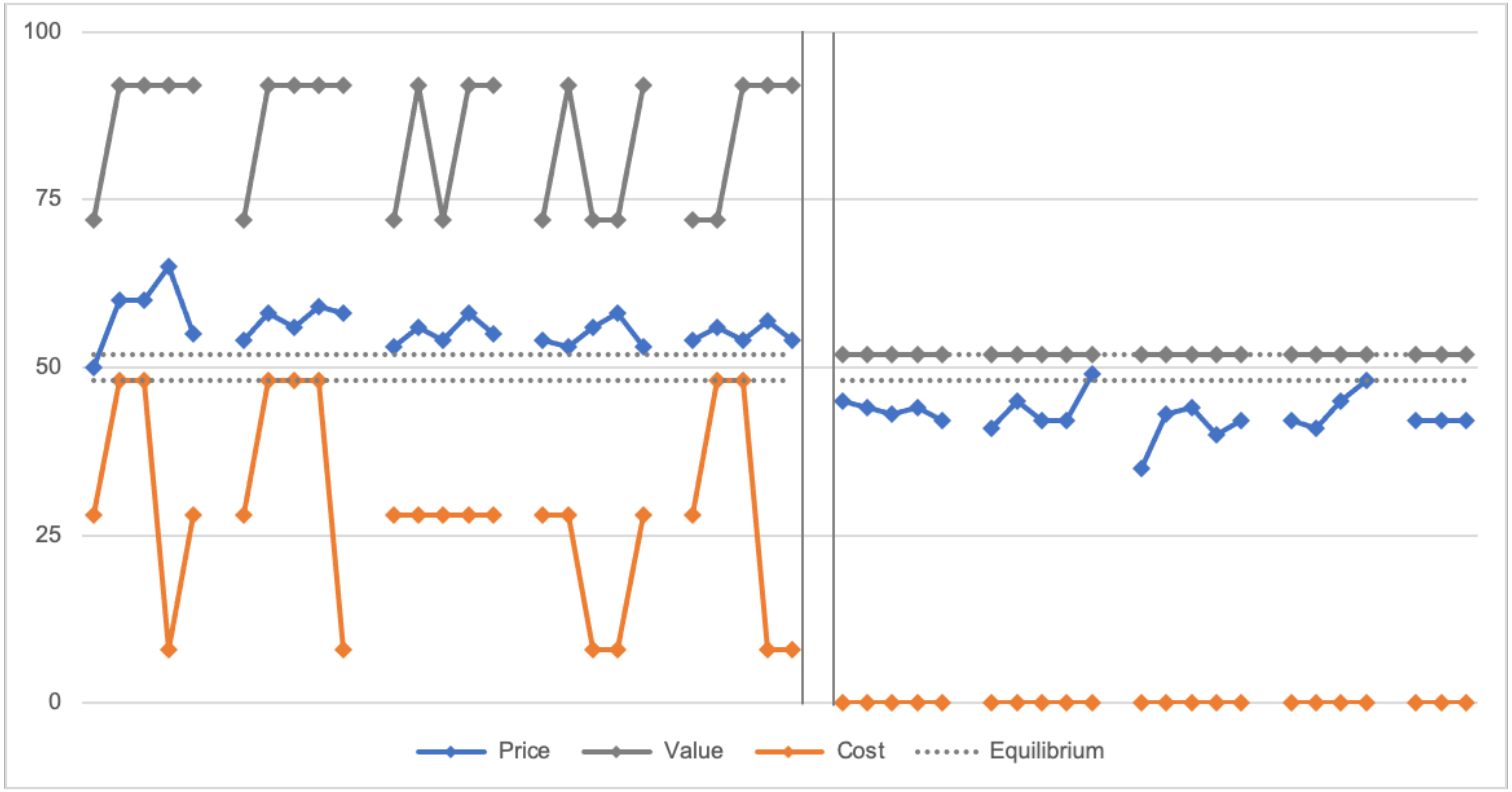
To get a handle on order effects, I conduct two sessions and vary the order the auctions.

Experimental design

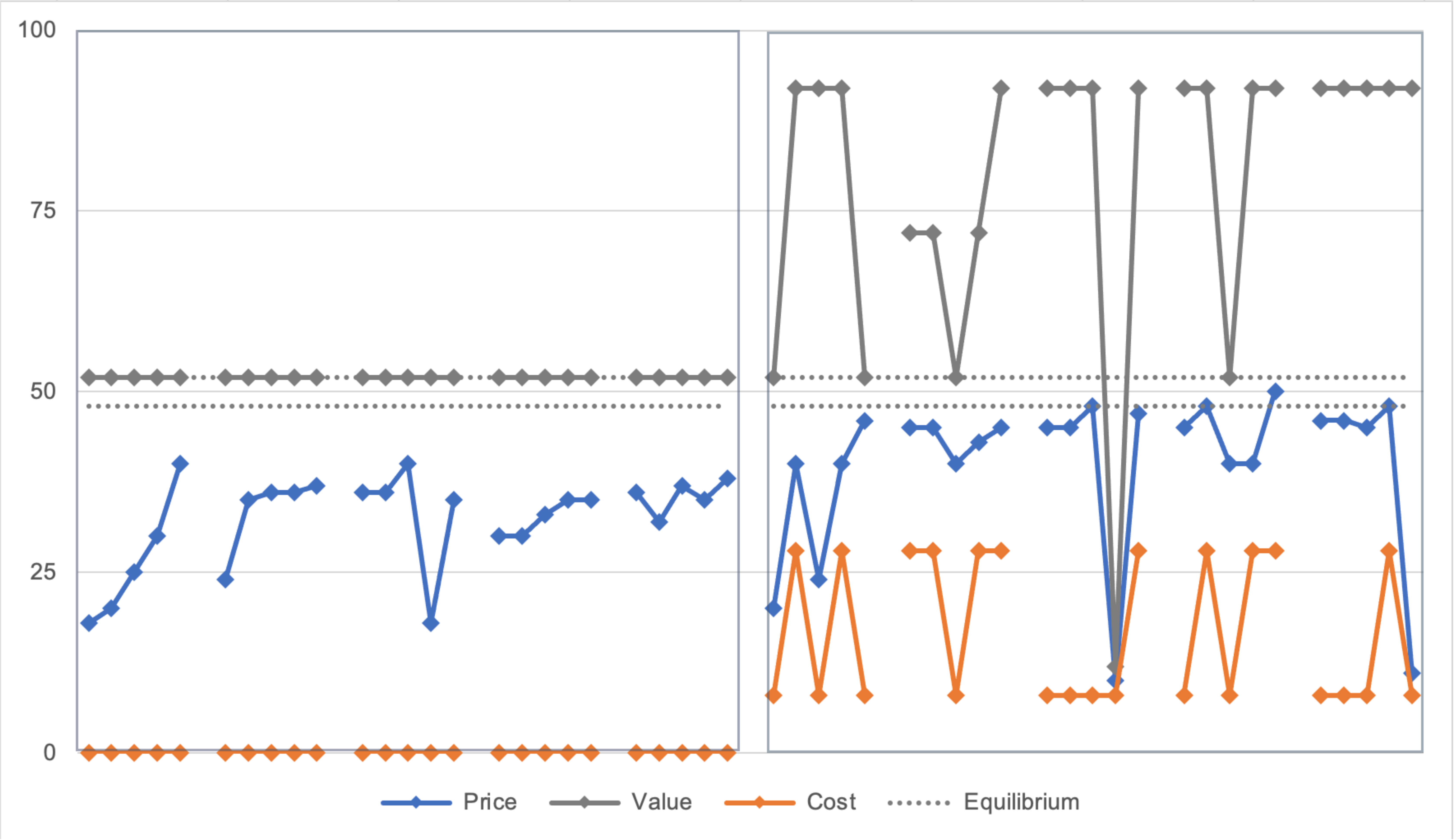
Experimental details:

- 5 active buyers and 5 active sellers, plus a further 8 in the queue (half buyers, half sellers). Each period ended once the queue was exhausted.
- The auction rules were presented in written form, and further emphasised through a quiz
- To make or accept an offer, subjects needed to raise their hand. All offers were repeated by the auctioneer and recorded on a whiteboard
- Auctions used a standard ‘improvement rule’
- In line with recent recommendations (Charness, Gneezy, and Halladay, 2016; Azrieli, Chambers, and Healy, 2018) and double auction experiments (Nax et al., 2020), subjects only paid for one randomly chosen round

Results



Results



Results

Result 1: shifting values/costs down lowers observed prices:

- Comparing the two sessions, we see that average prices are 56.0 (symmetric treatment) vs 32.3 (low value treatment) ($p < 0.0001$).
- In the first session, shifting values down reduces average bids from 56.0 to 42.9 ($p < 0.0001$).
- In the second session, shifting values up increases average bids from 32.3 to 40.0 ($p < 0.01$).

Results

Examining bids and asks reveals a similar pattern:

- Comparing the two sessions, we see that average bids/asks are 47.0/67.9 (symmetric treatment) vs 26.6/58.9 (low value treatment) ($p < 0.0001$, $p = 0.03$).
- In the first session, shifting values down reduces average bids from 47.0 to 38.0 ($p < 0.01$) and reduces average asks from 67.9 to 48.5 ($p < 0.0001$).
- In the second session, shifting values up increases average bids from 26.6 to 31.7 ($p = 0.03$) and increases average asks from 59.0 to 153.2 ($p = 0.02$).

Results

Result 2: prices are almost never at the CE (including in the symmetric treatments!)

- In the first session, prices start persistently above the CE, and then fall persistently below it. Moreover, we can easily reject CE plus noise ($p < 0.0001$, $p < 0.0001$), even choosing the closest CE price to stack the deck in CE's favour
- In the second session, prices are below the CE throughout (despite rising markedly in the second half). Again, we can easily reject CE plus noise ($p < 0.0001$, $p < 0.01$).

Results

Result 3: prices do not seem to be converging to the CE

- In the first experiment, the average changes are close to zero (0.17 and -0.14 in the first and second half respectively), and neither are statistically different from zero ($p = 0.84$, $p = 0.89$). Moreover, the CE does not seem to be an absorbing state
- In the second experiment, the average changes are again close to zero (0.83, 0.24) and again statistically insignificant ($p = 0.61$, $p = 0.94$). Moreover, the CE again does not seem to be very 'sticky'

Understanding the monotonicity

- It is not hard to find models that rationalise the observed monotonicity
- To start, consider Gode and Sunder (1993): buyers bid uniformly between 0 and their valuation, sellers bid uniformly between their valuation and some maximum, and trade occurs when the market bid and market ask 'cross' (at the earlier of the two bids).
- Under such assumptions, decrease values leads to stochastically higher bid distributions for both buyers and sellers; and so tends to push up prices.
- Indeed, a simulation reveals that ZI trading (with an upper bound of 100) generates average prices of about £50 in the symmetric treatment, and average prices of about £35 in the second treatment.

Understanding the monotonicity

- While postulating randomness does not exactly ‘explain’ anything, optimising models give rise to the same monotonicity
- Consider Gjerstad and Dickhaut (1995): buyers/sellers choose bids/asks to myopically maximise this period payoff. Ignoring integer constraints, the optimal bid satisfies a first order condition, inspection of which reveals that higher valuations lead to higher optimal bid /asks.
- This provides a second explanation for the monotonicity

Understanding the monotonicity

- Finally, consider Friedman (1991): buyers/sellers play aggressive reservation price strategies, where reservation prices are chosen to optimally balance the benefit of waiting for better bids/offers against the costs of running out of time
- As Friedman remarks, optimal reservation prices are monotone in valuations. For example, buyers with lower values are happy to accept lower offers. Therefore, increasing valuations will tend to drive up observed prices.

The Marshallian path

Although the previous sections demonstrate that double auctions need not generate CE, one might think that this has something to do with the 'queuing' system used to hold the CE fixed.

If instead players drop out of the auctions as trade progresses, one may suspect that prices approach to the CE due to a "Marshallian path" dynamic (see Brewer et al 2002 for informal discussion).

The Marshallian path

To understand this dynamic, return to the environment in Section 2 (recalling that F and G denote the distributions of buyer and seller valuations respectively, and p^* denotes the unique CE price.)

Consider now a sequence of trades, indexed by $t \in [0,T]$. Let $v_b(t)$, $v_s(t)$ and $p(t)$ denote the buyer valuation, seller valuation, and price associated with trade t; and (with some abuse of notation) denote the corresponding functions by v_b, v_s and p.

The Marshallian path

Definition 3. A *Marshallian path* is a triple (v_b, v_s, p) such that

1. For all $t \in [0,T]$, $v_b(t) = F^{-1}(1-t)$ and $v_s(t) = G^{-1}(t)$.
2. For all $i \in [0,1]$, $i \in [0,T]$ if and only if $F^{-1}(1-i) \geq G^{-1}(i)$.
3. For all $t \in [0,T]$, $v_s(t) \leq p(t) \leq v_b(t)$.

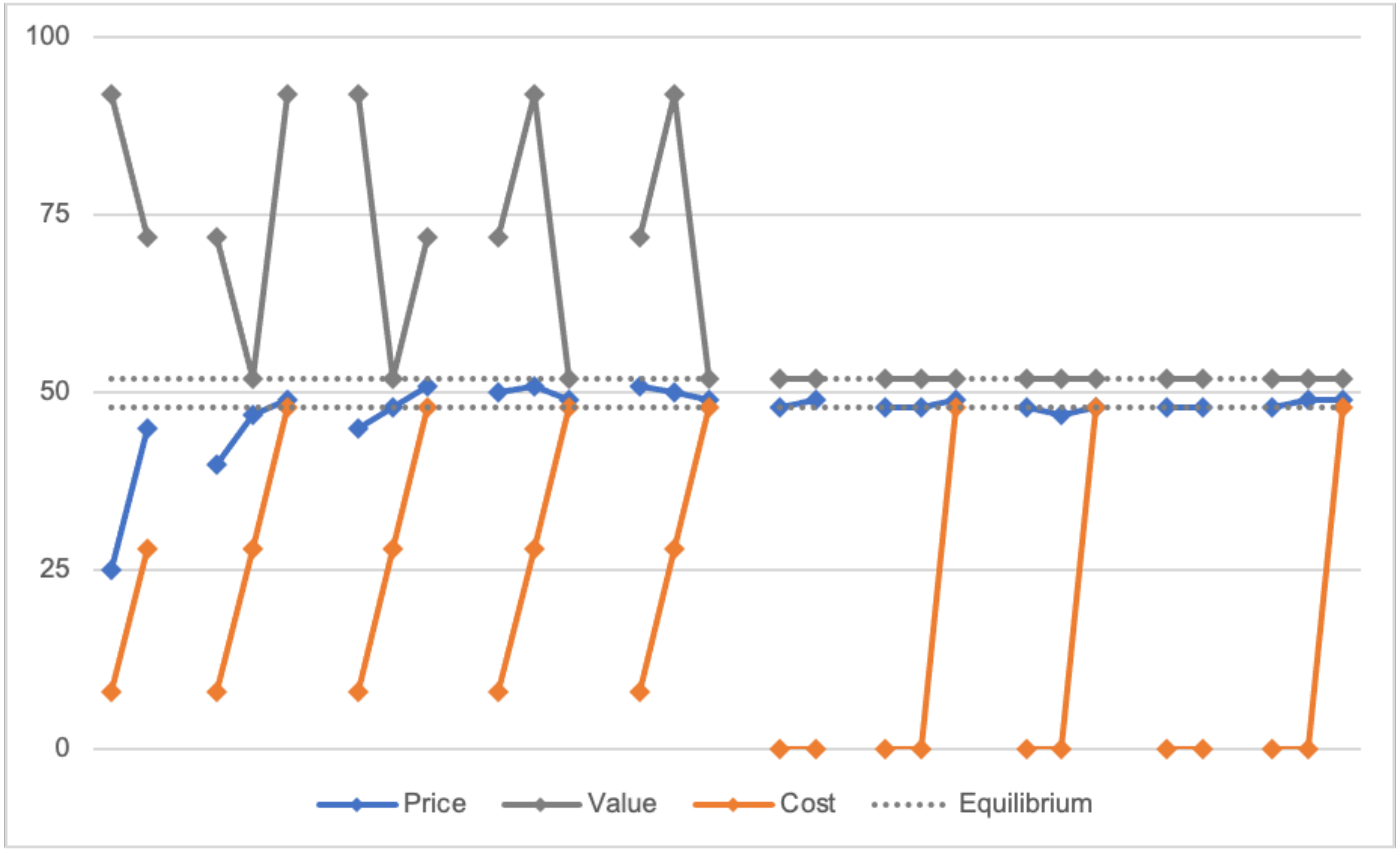
The Marshallian path

Proposition 2. If (v_b, v_s, p) is a Marshallian path, then $p(t) \in [F^{-1}(1-t), G^{-}1(t)]$ for all $t \in [0,T]$. As a result, $p(T) = p*$.

The Marshallian path

- Why should trade follow a Marshallian path? See Section 2!
- This motivates our final experiment. This is exactly the same as the previous experiment, except buyers/sellers can drop out as trade progresses.

The Marshallian path



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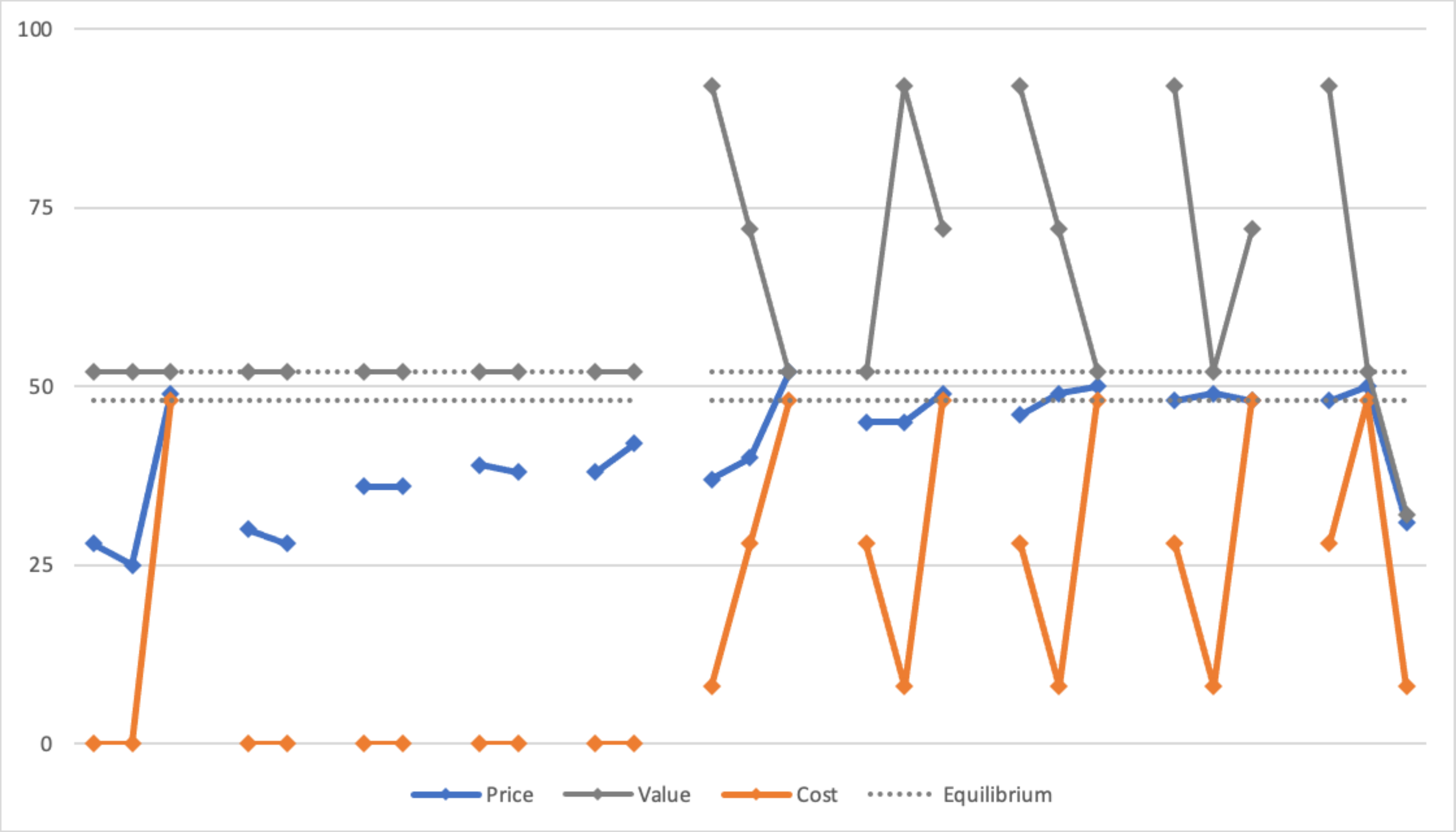

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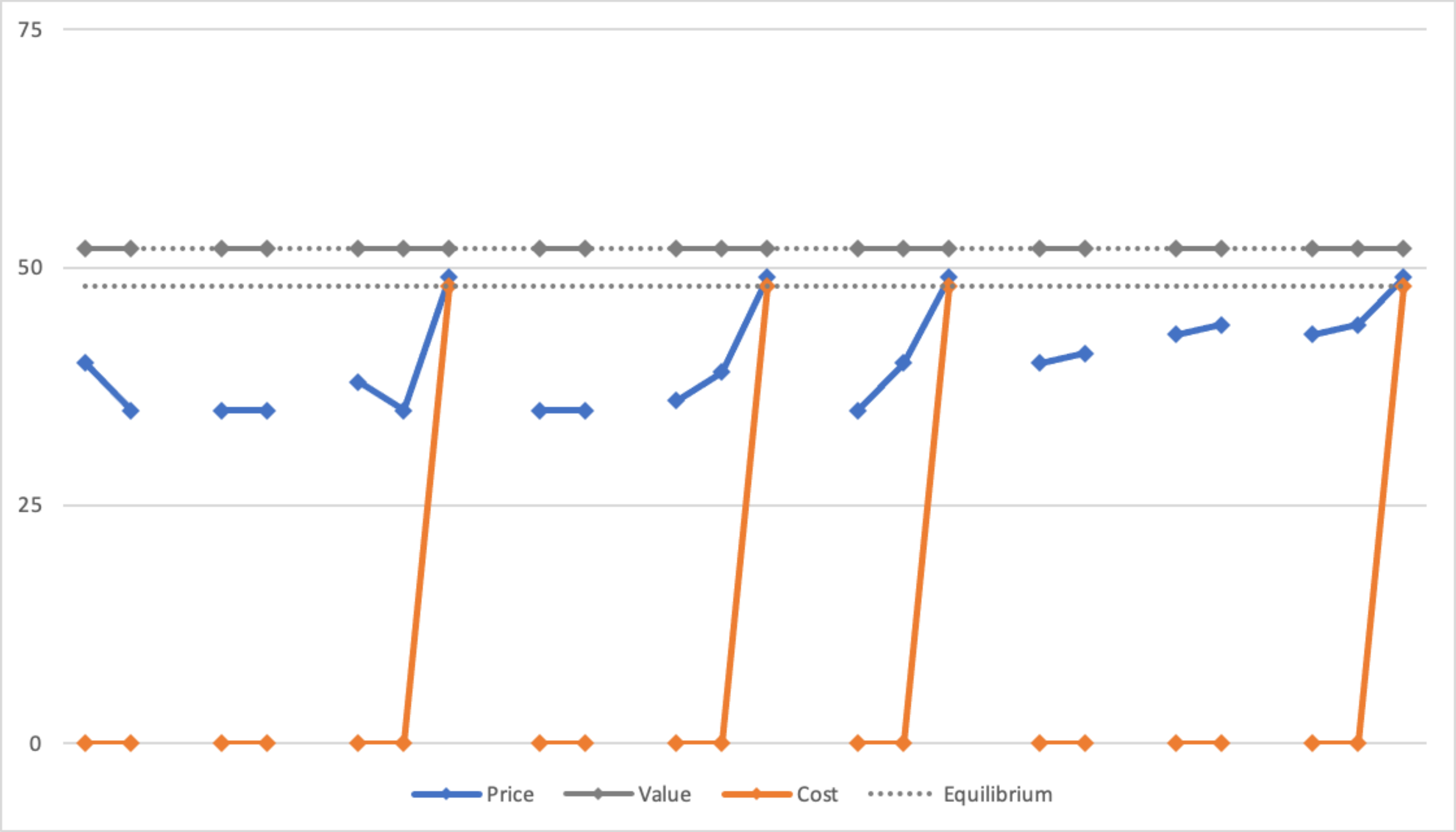

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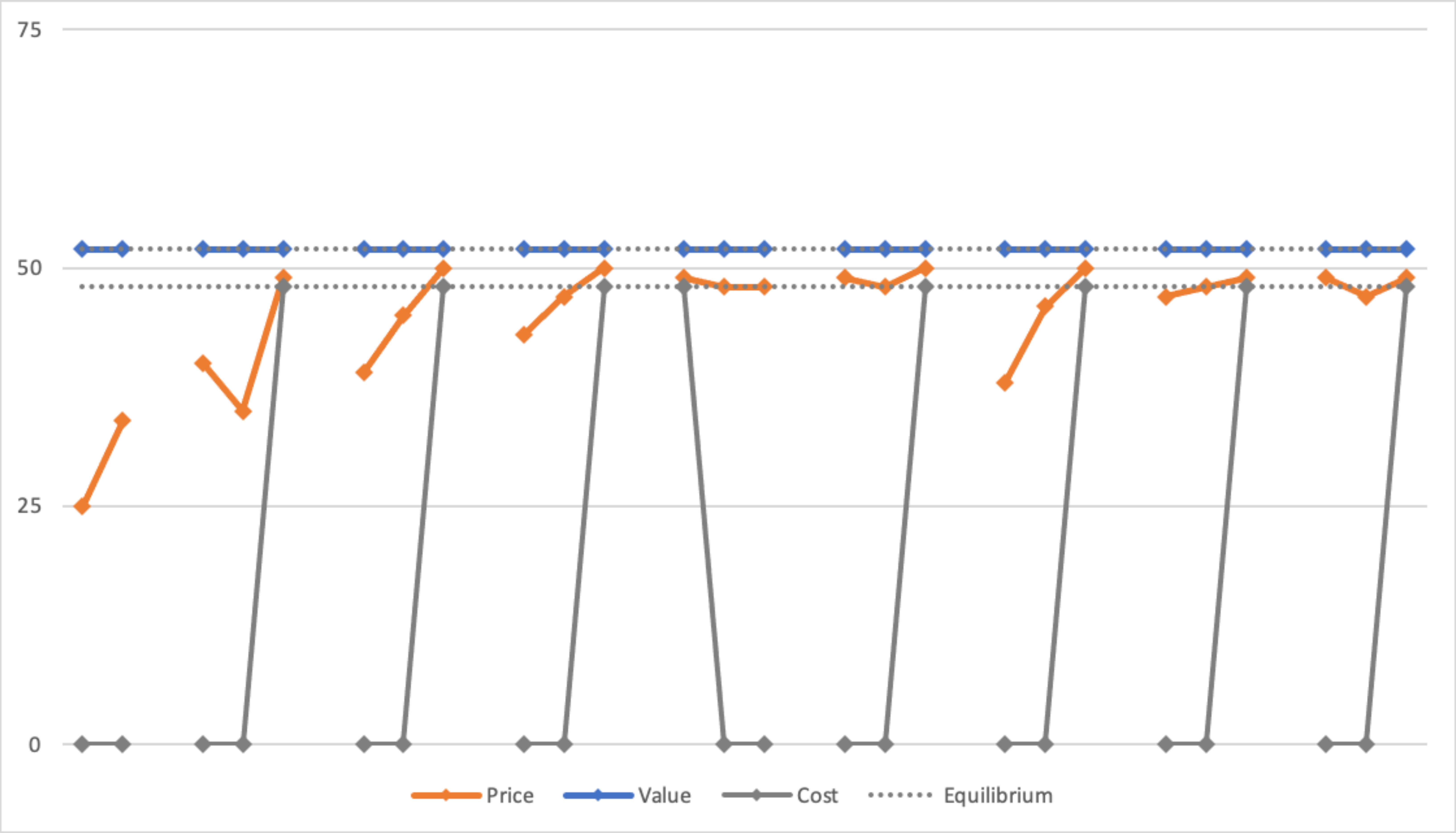

The Marshallian path

Sometimes the third transaction fails to occur. Mechanically, this seems to be due to reluctance on the part of buyers (see pictures).

Why are buyers reluctant to pay the equilibrium price?

1. Inequality aversion: buyers might think that sellers are demanding too much surplus (costs are private information)

2. Strategic considerations: by refusing to trade at the equilibrium price, buyers might be trying to get better prices in the next period (an irony…)

Conclusions

Main take-aways:

1. In environments with stationary value distributions, CE preserving shifts do indeed alter observe prices

2. However, the effect of these examples is somewhat blunted once traders are allowed to drop out

Taken together, these findings highlight the importance of the Marshallian path for understanding equilibration. As a result, they can shed light on the 'scientific mystery' introduced by Smith (1962) almost 60 years ago.